

# Strictly Anomaly Mediated Supersymmetry Breaking

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**ABSTRACT:** We consider an extension of the MSSM with anomaly mediation as the only source of supersymmetry-breaking, and the tachyonic slepton problem solved by a gauged  $U(1)$  symmetry. The extra gauge symmetry is broken at high energies in a manner preserving supersymmetry, while also introducing both the see-saw mechanism for neutrino masses, and the Higgs  $\mu$ -term. We call the model sAMSB (*strictly* anomaly mediated supersymmetry breaking).

We present typical spectra for the model and compare them with those from so-called *minimal* anomaly mediated supersymmetry breaking. We find a SM-like Higgs of mass 125 GeV with a gravitino mass of 140 TeV and  $\tan\beta = 16$ . However, the muon anomalous magnetic moment is  $3\sigma$  away from the experimental value.

The model naturally produces a period of hybrid inflation, which can exit to a false vacuum characterised by large Higgs vevs, reaching the true ground state after a period of thermal inflation. The scalar spectral index is reduced to approximately 0.975, and the correct abundance of neutralino dark matter can be produced by decays of thermally-produced gravitinos, provided the gravitino mass (and hence the Higgs mass) is high. Naturally light cosmic strings are produced, satisfying bounds from the Cosmic Microwave Background. The complementary pulsar timing and cosmic ray bounds require that strings decay primarily via loops into gravitational waves. Unless the loops are extremely small, the next generation pulsar timing array will rule out or detect the string-derived gravitational radiation background in this model.

**KEYWORDS:** [Supersymmetry](#), [anomaly mediation](#), [inflation](#), [cosmic strings](#).

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## 1. Introduction

The SM Higgs-like particle of mass 125GeV recently discovered at the LHC [1, 2] strongly constrains future model building, while recent negative results from both

the Tevatron and LHC in searches for sparticles place increasing pressure on models with low energy supersymmetry. Here we explore a specific supersymmetric model in which the low energy spectrum is that of the Minimal Supersymmetric Standard Model (MSSM), but the gauge symmetry is augmented by an extra gauged  $U(1)$  symmetry,  $U(1)'$ , spontaneously broken at high energies in a manner which affects both physics at the supersymmetry breaking scale and physics at high scales characterising inflation and cosmic strings.

The broad features of the model are independent of the source of supersymmetry breaking, but if we assume that this source is in fact anomaly mediation (AMSB) [3]-[5], then there arises an interesting interplay between the low energy physics (and in particular the Higgs  $\mu$ -term) and the high energy physics involving strings and inflation. Moreover the breaking of  $U(1)'$  solves the tachyonic slepton problem characteristic of AMSB [5, 6].

We first presented this specific model in [7], in a form where we also introduced a Fayet-Iliopoulos (FI) term for the  $U(1)'$ . Here we concentrate on the minimal formulation when there is no such term<sup>1</sup>. The model implements a form of AMSB which we refer to as *strictly* anomaly-mediated supersymmetry-breaking (sAMSB), by which we mean that there are no other sources of supersymmetry breaking beside the F-term of the conformal compensator field. As a consequence the soft parameters have an elegant renormalisation group (RG) invariant form. It therefore differs from so-called minimal AMSB(mAMSB), which posits an extra source of supersymmetry-breaking, instead of extra fields, in order to solve the tachyonic slepton problem. Our model is not quite a complete sAMSB implementation, in that it requires an extension to determine the soft parameter associated with the Higgs  $\mu$ -term.

We begin by describing the symmetries and field content of the model and explaining in detail how the spontaneous breakdown of the  $U(1)'$  symmetry at a large scale  $M$  not only solves the AMSB tachyonic slepton problem, but also generates a Higgs  $\mu$  term and the see-saw mechanism for neutrino masses. This outcome is achieved by the introduction of three new chiral superfields;  $S$ , which is a gauge singlet and a pair of  $SU(3) \otimes SU(2) \otimes U(1)_Y$  singlet fields  $\Phi, \bar{\Phi}$  which are oppositely charged under  $U(1)'$ . We then exhibit characteristic sparticle spectra for the model; the calculations involved to obtain these are essentially as described in Refs. [9, 10], but allowing for a larger gravitino mass. We also discuss the fine-tuning issue raised by this, and compare the results of our model with results from the most popular (but, we will argue, less elegant) version of AMSB, generally called mAMSB. We will see that sAMSB generally

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<sup>1</sup>Aside from the fact that this makes the model more appealing by removing an independent mass scale, we also thereby avoid confrontation with the conclusion of Komargodski and Seiberg [8] that a global theory with a FI term cannot be consistently embedded in supergravity

keeps sleptons lighter than in mAMSB, which means that the contribution to the muon anomalous magnetic moment  $a_\mu \equiv (g_\mu - 2)/2$  is typically higher for a given Higgs mass.

The theory incorporates a natural mechanism for supersymmetric F-term inflation, with the scalar component of  $S$  as the inflaton. Previously, we concentrated on a region of parameter space such that inflation ended with a transition to a state with only the  $U(1)'$  broken. There is, however, an interesting alternative that inflation ends with the development of vevs for the Higgs multiplets,  $h_{1,2}$ , breaking the electroweak symmetry. A combination of the Higgs fields  $h_1 \cdot h_2$  and the scalar components of the singlet fields  $\phi\bar{\phi}$  is a flat direction, lifted by soft supersymmetry-breaking terms, and the normal low energy electroweak vacuum is achieved after a later period of thermal inflation. Approximately 17 e-foldings of thermal inflation reduce the number of e-foldings of high scale inflation, and therefore reduce the spectral index of scalar Cosmic Microwave Background (CMB) fluctuations to about 0.975. This is within about  $1\sigma$  of the WMAP7 value.

The reheat temperature after this period of thermal inflation is around  $10^9$  GeV, which means that there is no gravitino problem: gravitinos are very massive, more than 40 TeV, and so decay early enough not to be in conflict with nucleosynthesis. Indeed, the gravitino problem can turn into the gravitino solution for the typical AMSB feature of too low a dark matter density generated at freeze-out: the lightest supersymmetric particle (LSP) is mostly wino and has a relatively high annihilation cross-section. In our model a critical density of LSPs can be generated by gravitino decays, if the gravitino (and hence the LSP) is heavy enough.

The model also has the possibility of baryogenesis via leptogenesis following thermal inflation, with CP violation supplied by the neutrino sector. The field giving mass to right-handed neutrinos has an inflation-scale ( $10^{16}$  GeV) vacuum expectation value (vev), but if the lightest right-handed neutrino is sufficiently light to be generated at reheating after thermal inflation, a lepton asymmetry can be generated by its out-of-equilibrium decay.

There is a broken  $U(1)$  symmetry in the model, and cosmic strings with a  $10^{16}$  GeV mass scale are formed, although not until the end of thermal inflation. There is a large Higgs condensate in the core of the string which spreads the string out to a width of order the supersymmetry-breaking scale, and reduces its mass per unit length by well over an order of magnitude. The strings satisfy CMB constraints on the mass per unit length from combined WMAP7 and small-scale observations. Their decays are constrained by pulsar timing observations in the case of gravitational waves and the diffuse  $\gamma$ -ray background in the case of particle production: the latter means that less than about 0.1% of the energy of the strings should end up as particles, and the former puts constraints on the average size of the loops at formation.

The model has the same field content as the  $F_D$  hybrid inflation model [11, 12], but different charge assignments and couplings.  $F_D$  hybrid inflation also has a singlet which is a natural inflaton candidate, but differs in other ways: for example, right-handed neutrinos have electroweak-scale masses, and the gravitino problem is countered by entropy generation.

The model also has the same field content as the B–L model of Refs. [13], although as explained in section 3, the  $U(1)'$  symmetry cannot be  $U(1)_{B-L}$  in AMSB. It is also closely related to the model of Ref. [14], in which the fields  $\Phi, \bar{\Phi}$  are  $SU(2)_R$  triplets. This also has a flat direction involving the Higgs, although the authors did not pursue its consequences.

To summarise our results: at  $\tan \beta = 10$ , sAMSB can accommodate a Higgs mass above 120 GeV for gravitino masses over 80 TeV, while accounting for the discrepancy in  $a_\mu$  between the Standard Model (SM) theory and experiment to within  $2\sigma$  would have favoured 80 TeV or lower. Larger values of  $\tan \beta$  allow a more massive Higgs: for  $\tan \beta = 16$  we find a Higgs mass of 125 GeV for a gravitino mass of 140 TeV.

sAMSB also allows for an observationally consistent dark matter density, if the gravitino mass is over about 100 TeV, with the dark matter deriving from the decay of gravitinos produced from reheating after thermal inflation. The spectral index of scalar cosmological perturbations is within  $1\sigma$  of the WMAP7 value, and the observational bounds on cosmic strings can be satisfied if the strings decay into gravitational radiation. The model has also has a natural mechanism for baryogenesis via leptogenesis through the decays of right-handed neutrinos.

## 2. The AMSB soft terms

We will assume that supersymmetry breaking arises via the renormalisation group invariant form characteristic of Anomaly Mediation, so that the soft parameters for the gaugino mass  $M$ , the  $\phi^3$  interaction  $h$  and the  $\phi^*\phi$  and  $\phi^2$  mass terms  $m^2$  and  $m_3^2$  in the MSSM take the generic RG invariant form

$$M_i = m_{\frac{3}{2}} \beta_{g_i} / g_i \quad (2.1)$$

$$h_{U,D,E,N} = -m_{\frac{3}{2}} \beta_{Y_{U,D,E,N}} \quad (2.2)$$

$$(m^2)^i_j = \frac{1}{2} m_{\frac{3}{2}}^2 \mu \frac{d}{d\mu} \gamma^i_j \quad (2.3)$$

$$m_3^2 = \kappa m_{\frac{3}{2}} \mu_h - m_{\frac{3}{2}} \beta_{\mu_h}. \quad (2.4)$$

Here  $\mu$  is the renormalisation scale, and  $m_{\frac{3}{2}}$  is the gravitino mass;  $\beta_{g_i}$  are the gauge  $\beta$ -functions and  $\gamma$  is the chiral supermultiplet anomalous dimension matrix.  $Y_{U,D,E,N}$

are the  $3 \times 3$  Yukawa matrices, and  $\mu_h$  is the superpotential Higgs  $\mu$ -term. We will see that in our low energy theory, Eq. (2.3) is replaced, in fact, by

$$(m^2)^i{}_j = \frac{1}{2} m_{\frac{3}{2}}^2 \mu \frac{d}{d\mu} \gamma^i{}_j + k Y_i \delta^i{}_j, \quad (2.5)$$

where the  $Y_i$  are charges corresponding to a  $U(1)$  symmetry. This  $kY$  term corresponds in form to the contribution of an FI D-term, and can be employed to obviate the tachyonic scalar problem characteristic of AMSB. How such a term can be generated (with AMSB) was first discussed in Ref. [5], and first applied to the MSSM in Ref. [6]. The basic idea was pursued in a number of papers [15]-[19]. For example, Ref. [15] demonstrated explicitly the UV insensitivity of the result, and Ref. [16] emphasised that the tachyonic problem could be solved using a single  $U(1)$  rather than a linear combination of two, the approach followed in Ref. [6]. An extension of the MSSM such that the spontaneous breaking of a gauged  $U(1)'$  with an FI term gave rise to the  $kY$  term was written down in Ref. [9]. In [7] we developed an improved version of this model, retaining the possibility of a *primordial* FI term for  $U(1)'$ ; here we will dispense with the FI term, and emphasise that we can nevertheless generate the  $kY$  term naturally with  $k$  of  $O(m_{\frac{3}{2}}^2)$ , by breaking a  $U(1)'$  symmetry at a large scale, *without introducing an explicit FI term*.

At first sight Eq. (2.5) resembles the formula for the scalar masses employed in the so-called mAMSB model, where the  $kY_i$  term is replaced by a universal scalar mass contribution  $m_0^2$ . The differences are as follows:

- The mAMSB involves the introduction of an additional source of supersymmetry breaking independent of the gravitino mass, while, as we shall see, Eq. (2.5) does not.
- The parameter  $k$  in Eq. (2.5) turns out to be more constrained than  $m_0^2$ . This is associated with the fact that inevitably all the  $Y_i$  cannot have the same sign.
- The elegant RG invariance of Eq. (2.1)-Eq. (2.4) is preserved by Eq. (2.5).

It is these observations that prompts us to refer to our model as sAMSB. Note that, of course, we cannot “promote” the mAMSB into the sAMSB by the addition of additional heavier fields which cancel the associated  $U(1)'$  anomaly; with an unbroken  $U(1)'$ , any massive chiral multiplets will obviously make no contribution to this anomaly.

Eq. (2.4) is the most general form for  $m_{\frac{3}{2}}^2$  that is consistent with RG invariance, as first remarked explicitly in Ref. [9]; the parameter  $\kappa$  is an arbitrary constant. For

	$Q$	$U$	$D$	$H_1$	$H_2$	$N$
$q$	$-\frac{1}{3}q_L$	$-q_E - \frac{2}{3}q_L$	$q_E + \frac{4}{3}q_L$	$-q_E - q_L$	$q_E + q_L$	$-2q_L - q_E$

**Table 1:** Anomaly free U(1) symmetry for arbitrary lepton doublet and singlet charges  $q_L$  and  $q_E$  respectively.

discussion of possible origins of  $m_{\frac{3}{2}}^2$  from the underlying superconformal calculus formulation of supergravity see Refs. [5, 18, 19]. We will simply assume that the model can be generalised to produce such a term; the procedure which has, in fact, been generally followed. The presence of  $\kappa$  means that in sparticle spectrum calculations one is free to calculate  $m_{\frac{3}{2}}^2$  (and the value of the Higgs  $\mu$ -term,  $\mu_h$ ) by minimising the Higgs potential at the electroweak scale in the usual way. (For  $\kappa = 1$ , which is the value suggested by a straightforward use of the conformal compensator field [5], one might have hoped to use the minimisation conditions to determine  $\tan\beta$ , but it turns out this leads to a very small value of  $\tan\beta$  incompatible with gauge unification, because of the correspondingly large top Yukawa coupling [17]). We will see, however, that in our model the result for  $\mu_h$  has implications for other parameters in the *underlying* theory which are constrained by cosmological considerations.

### 3. The U(1)' symmetry

The MSSM (including right-handed neutrinos) admits two independent generation-blind anomaly-free U(1) symmetries. The possible charge assignments are shown in Table 1. The SM gauged U(1)<sub>Y</sub> is  $q_L = -1, q_E = 2$ ; this U(1) is of course anomaly free even in the absence of  $N$ . U(1)<sub>B-L</sub> is  $q_E = -q_L = 1$ ; in the absence of  $N$  this would have U(1)<sup>3</sup> and U(1)-gravitational anomalies, but no mixed anomalies with the SM gauge group.

Our model will have, in addition, a pair of MSSM singlet fields  $\Phi, \bar{\Phi}$  with U(1)' charges  $q_{\Phi, \bar{\Phi}} = \pm(4q_L + 2q_E)$  and a gauge singlet  $S$ . In order to solve the tachyon slepton problem we will need that, for our new gauge symmetry U(1)', the charges  $q_L, q_E$  have the same sign at low energies. As explained in Ref. [10], however, it is in fact more appropriate to input parameters at high energies, when in fact although necessarily  $q_E > 0$ , the range of acceptable values of  $q_L$  includes negative ones; not negative enough, however, to allow U(1)' to be U(1)<sub>B-L</sub>.

Thus sAMSB has three input parameters  $m_{\frac{3}{2}}, kq_L, kq_E$ , associated with the supersymmetry breaking sector, while mAMSB only has two:  $m_{\frac{3}{2}}, m_0$ . However, it turns out

that because the allowed  $(q_L, q_E)$  region is so restricted, sAMSB is the more predictive of the two. We will see this explicitly in section 6.

#### 4. The superpotential and spontaneous $U(1)'$ breaking

The complete superpotential for our model is:

$$W = W_A + W_B \quad (4.1)$$

where  $W_A$  is the MSSM superpotential, omitting the Higgs  $\mu$ -term, and augmented by Yukawa couplings for the right-handed neutrinos,  $N$ :

$$W_A = H_2 Q Y_U U + H_1 Q Y_D D + H_1 L Y_E E + H_2 L Y_N N \quad (4.2)$$

and

$$W_B = \lambda_1 \Phi \bar{\Phi} S + \frac{1}{2} \lambda_2 N N \Phi - \lambda_3 S H_1 H_2 - M^2 S, \quad (4.3)$$

where  $M, \lambda_1, \lambda_3$  are real and positive and  $\lambda_2$  is a symmetric  $3 \times 3$  matrix. The sign of the  $\lambda_3$  term above is chosen because with our conventions, in the electroweak vacuum where

$$H_1 = \left( \frac{v_1}{\sqrt{2}}, 0 \right)^T \quad \text{and} \quad H_2 = \left( 0, \frac{v_2}{\sqrt{2}} \right)^T \quad (4.4)$$

we have  $H_1 H_2 \rightarrow -\frac{1}{2} v_1 v_2$ .

The  $U(1)'$  symmetry forbids the renormalisable  $B$  and  $L$  violating superpotential interaction terms of the form  $QLD, UDD, LLE, H_1 H_2 N, NS^2, N^2 S$  and  $N^3$ , as well as the mass terms  $NS, N^2$  and  $LH_2$  and the linear term  $N$ . Moreover  $W_B$  contains the only cubic term involving  $\Phi, \bar{\Phi}$  that is allowed. Our superpotential Eq. (4.1) is completely natural, in the sense that it is invariant under a global  $R$ -symmetry, with superfield charges

$$S = 2, L = E = N = U = D = Q = 1, H_1 = H_2 = \Phi = \bar{\Phi} = 0, \quad (4.5)$$

which forbids the remaining gauge invariant renormalisable terms ( $S^2, S^3, \Phi \bar{\Phi}$  and  $H_1 H_2$ ). This  $R$ -symmetry also forbids the quartic superpotential terms  $QQQL$  and  $UUDE$ , which are allowed by the  $U(1)'$  symmetry, and give rise to dimension 5 operators capable of causing proton decay [20]-[22]. It is easy to see, in fact, that the charges in Eq. (4.5) disallow  $B$ -violating operators in the superpotential of arbitrary dimension. Of course this  $R$ -symmetry is broken by the soft supersymmetry breaking.



## 5. The Higgs potential

In this section we discuss the spontaneous breaking of the  $U(1)'$  symmetry and its consequences. We shall assume  $M$  is much larger than the scale of supersymmetry-breaking. (Such a large tadpole term has been disfavoured in the past; moreover it has been argued that it would generally be expected to lead to a large vev  $\langle s \rangle$ , but as we shall see this does not happen in our model.) It is then clear from the form of the superpotential  $W_B$  as given in Eq. (4.3) that for an extremum that is supersymmetric (when we neglect supersymmetry-breaking) we will require non-zero vevs for  $\phi, \bar{\phi}$  and/or  $h_{1,2}$  (in order to obtain  $F_S = 0$ ). The existence of competing vacua of this nature was noted in by Dvali et al in Ref. [14]; their model differs from ours in choice of gauge group (they have  $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ ) and supersymmetry-breaking mechanism.

Let us consider these two possibilities in turn.

### 5.1 The $\phi, \bar{\phi}, s$ extremum

Retaining for the moment only the scalar fields  $\phi, \bar{\phi}, s$  (the scalar component of their upper case counterpart superfields) we write the scalar potential:

$$\begin{aligned} V = & \lambda_1^2 (|\phi s|^2 + |\bar{\phi} s|^2) + |\lambda_1 \phi \bar{\phi} - M^2|^2 + \frac{1}{2} q_{\Phi}^2 g'^2 (|\phi|^2 - |\bar{\phi}|^2)^2 \\ & + m_{\phi}^2 |\phi|^2 + m_{\bar{\phi}}^2 |\bar{\phi}|^2 + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) \\ & + h_{\lambda_1} \phi \bar{\phi} s + c.c.. \end{aligned} \quad (5.1)$$

Here, as well as soft terms dictated by Eqs. (2.2), (2.3), we also introduce a soft breaking term linear in  $s$ . (In fact, according to Ref. [23], for a nonvanishing  $RG$  invariant form of  $\rho$  we would require a *quadratic* term in  $s$  in the superpotential, which in fact we do not have. We nevertheless consider the possible impact of a  $\rho$  term, but will presently assume it is small, even if non-zero).

The potential depends on two explicit mass parameters, the gravitino mass  $m_{\frac{3}{2}}$  and  $M$ . Let us establish its minimum. Writing  $\langle \phi \rangle = v_{\phi}/\sqrt{2}$ ,  $\langle \bar{\phi} \rangle = v_{\bar{\phi}}/\sqrt{2}$  and  $\langle s \rangle = v_s/\sqrt{2}$ , we find

$$v_{\phi} \left[ m_{\phi}^2 + \frac{1}{2} \lambda_1^2 v_s^2 + \frac{1}{2} g^2 q_{\Phi}^2 (v_{\phi}^2 - v_{\bar{\phi}}^2) \right] + v_{\bar{\phi}} \left[ \lambda_1 \left( \frac{1}{2} \lambda_1 v_{\phi} v_{\bar{\phi}} - M^2 \right) + \frac{h_{\lambda_1}}{\sqrt{2}} v_s \right] = 0 \quad (5.2)$$

$$v_{\bar{\phi}} \left[ m_{\bar{\phi}}^2 + \frac{1}{2} \lambda_1^2 v_s^2 - \frac{1}{2} g^2 q_{\Phi}^2 (v_{\phi}^2 - v_{\bar{\phi}}^2) \right] + v_{\phi} \left[ \lambda_1 \left( \frac{1}{2} \lambda_1 v_{\phi} v_{\bar{\phi}} - M^2 \right) + \frac{h_{\lambda_1}}{\sqrt{2}} v_s \right] = 0 \quad (5.3)$$

$$v_s \left[ m_s^2 + \frac{1}{2} \lambda_1^2 (v_{\phi}^2 + v_{\bar{\phi}}^2) \right] + \frac{h_{\lambda_1}}{\sqrt{2}} v_{\phi} v_{\bar{\phi}} + \sqrt{2} \rho M^2 m_{\frac{3}{2}} = 0. \quad (5.4)$$

It follows easily from Eqs. (5.2),(5.3) that

$$\lambda_1 \left( \frac{1}{2} \lambda_1 v_\phi v_{\bar{\phi}} - M^2 \right) = - \frac{v_\phi v_{\bar{\phi}}}{v_\phi^2 + v_{\bar{\phi}}^2} \left[ m_\phi^2 + m_{\bar{\phi}}^2 + \lambda_1^2 v_s^2 \right] - \frac{h_{\lambda_1}}{\sqrt{2}} v_s \quad (5.5)$$

$$\frac{1}{2} g'^2 q_\Phi^2 (v_\phi^2 - v_{\bar{\phi}}^2) = \frac{v_\phi^2 m_{\bar{\phi}}^2 - v_{\bar{\phi}}^2 m_\phi^2 + (v_\phi^2 - v_{\bar{\phi}}^2) \frac{1}{2} \lambda_1^2 v_s^2}{v_\phi^2 + v_{\bar{\phi}}^2}. \quad (5.6)$$

We now assume that  $M \gg m_{\frac{3}{2}}$ . It is immediately clear from Eqs. (5.5),(5.6) that

$$v_\phi^2 \simeq v_{\bar{\phi}}^2 \simeq \frac{2}{\lambda_1} M^2 \quad (5.7)$$

and then from Eq. (5.4) that  $v_s$  is  $O(m_{\frac{3}{2}})$ . We thus obtain from Eq. (5.6) that

$$v_\phi^2 - v_{\bar{\phi}}^2 = \frac{m_{\bar{\phi}}^2 - m_\phi^2}{g'^2 q_\Phi^2} + O(m_{\frac{3}{2}}^4/M^2) \quad (5.8)$$

and from Eq. (5.4) that

$$v_s = - \frac{h_{\lambda_1}}{\sqrt{2} \lambda_1^2} - \frac{m_{\frac{3}{2}} \rho}{\sqrt{2} \lambda_1} + O(m_{\frac{3}{2}}^2/M). \quad (5.9)$$

Now the  $h_{\lambda_1}$  term is determined in accordance with Eq. (2.2):

$$h_{\lambda_1} = -m_{\frac{3}{2}} \frac{\lambda_1}{16\pi^2} \left( 3\lambda_1^2 + \frac{1}{2} \text{Tr} \lambda_2^2 + 2\lambda_3^2 - 4q_\phi^2 g'^2 \right), \quad (5.10)$$

denoting the  $U(1)'$  charge by  $g'$ .

If we assume that  $|q_\Phi g'| \gg |\lambda_{1,2,3}|$  then we find

$$v_s \simeq - \frac{\sqrt{2} \lambda_1 m_{\frac{3}{2}}}{16\pi^2} \left( \frac{2q_\phi^2 g'^2}{\lambda_1^2} \right) - \frac{m_{\frac{3}{2}} \rho}{\sqrt{2} \lambda_1}. \quad (5.11)$$

For simplicity we shall assume that  $|\rho| \ll q_\phi^2 g'^2/(16\pi^2)$ , so that the  $\rho$  contributions to Eq. (5.9) and Eq. (5.11) are negligible.

Substituting back from Eqs. (5.7),(5.9) into Eq. (5.1), we obtain to leading order

$$V_\phi = \frac{1}{\lambda_1} M^2 \left( m_\phi^2 + m_{\bar{\phi}}^2 - \frac{h_{\lambda_1}^2}{2\lambda_1^2} \right). \quad (5.12)$$

Presently we shall compare this result with the analagous one associated with the  $h_{1,2}, s$  extremum.

Supposing, however, that the  $\phi, \bar{\phi}, s$  extremum is indeed the relevant one, we obtain the Higgs  $\mu$ -term

$$\mu_h = \frac{\lambda_1 \lambda_3 m_{\frac{3}{2}}}{16\pi^2} \left( \frac{2q_\phi^2 g'^2}{\lambda_1^2} \right). \quad (5.13)$$

One might think that since  $v_s$  is naturally determined above to be associated with the susy breaking scale (rather than the  $U(1)'$  breaking scale) it would be necessary to minimise the whole Higgs potential (including  $\langle h_{1,2} \rangle$ ) in order to determine it. But if we retain, for example, the  $m_s^2$  term in Eq. (5.4), the resulting correction to Eq. (5.9) is easily seen to be  $O(m_{\frac{3}{2}}^4/M^2)$ . Similarly, the Higgs vevs responsible for electroweak symmetry breaking do not affect Eqs. (5.9), (5.13) to an appreciable extent.

In Ref. [7], we naively estimated  $\mu_h \sim \lambda_1 \lambda_3 m_{\frac{3}{2}}$ , concluding that  $\mu_h$  would be at most  $O(\text{GeV})$  rather than  $O(100\text{GeV})$ . The improved formula Eq. (5.13) changes this conclusion.

If we neglect terms of  $O(m_{\frac{3}{2}})$ , it is easy to see from Eqs. (5.5), (5.6) that the breaking of  $U(1)'$  preserves supersymmetry (since in this limit the two equations correspond to vanishing of the  $S$  F-term and the  $U(1)'$  D-term respectively); thus the  $U(1)'$  gauge boson, its gaugino (with one combination of  $\psi_{\phi, \bar{\phi}}$ ) and the Higgs boson form a massive supermultiplet with mass  $m \sim g' \sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$ , while the remaining combination of  $\phi$  and  $\bar{\phi}$  and the other combination of  $\psi_{\phi, \bar{\phi}}$  form a massive chiral supermultiplet, with mass  $m \sim \lambda_1 \sqrt{v_\phi^2 + v_{\bar{\phi}}^2}$ .

For large  $M$ , all trace of the  $U(1)'$  in the effective low energy Lagrangian disappears, except for contributions to the masses of the matter fields, arising from the  $U(1)'$  D-term, which are naturally of the same order as the AMSB ones. Evidently  $S$  also gets a large supersymmetric mass, as does the  $N$  triplet, thus naturally implementing the see-saw mechanism. The generation of an appropriate  $\mu$ -term via the vev of a singlet is reminiscent of the NMSSM (for a review of and references for the NMSSM see Ref. [24]). We stress, however, that our model differs in a crucial way from the NMSSM, in that the low energy spectrum is precisely that of the MSSM.

It is easy to show by substituting Eq. (5.8) back into the potential, Eq. (5.1) that the contribution to the slepton masses arising from the  $U(1)'$  term which resolves the tachyonic slepton problem is given by

$$\delta m_{l,e}^2 \sim \frac{q_{L,E}}{2q_\Phi} (m_\phi^2 - m_{\bar{\phi}}^2) \quad (5.14)$$

with corresponding contributions for the other scalar MSSM fields proportional to their

$U(1)'$  charges. Now

$$m_\phi^2 - m_\phi^2 = \frac{1}{2} m_{\frac{3}{2}}^2 \mu \frac{d}{d\mu} (\gamma_{\bar{\phi}} - \gamma_\phi) = -\frac{1}{2} \frac{m_{\frac{3}{2}}^2}{16\pi^2} \text{Tr } \lambda_2 \beta_{\lambda_2} \quad (5.15)$$

where (at one loop)

$$16\pi^2 \beta_{\lambda_2} = \lambda_2 \left[ \lambda_1^2 + 2\lambda_2^2 + \frac{1}{2} \text{Tr } \lambda_2^2 + 2Y_N^\dagger Y_N - (2q_\phi^2 + 4q_N^2)g'^2 \right] \quad (5.16)$$

and we have for simplicity taken  $\lambda_2$  to be diagonal.

Let us consider what sort of values of  $\delta m_{l,e}^2$  we require. In this context it is interesting to compare Fig. 1 of Ref. [9] with Fig. 1 of Ref. [10]. In both references,  $(L, e)$  correspond to our  $(\delta m_l^2, \delta m_e^2)$  respectively. In the former case the scalar masses are calculated at low energies, whereas in the latter they are calculated at gauge unification and then run down to the electroweak scale. This is why the allowed  $(L, e)$  regions are different in the two cases. Since we are assuming  $M$  is large, it is clear that the latter are more relevant to our situation. From Fig. 1 of Ref. [10] we see that suitable values would be

$$\delta m_l^2 \simeq 0, \quad 0.16 \left( \frac{m_{\frac{3}{2}}}{40} \right)^2 \lesssim \delta m_e^2 \lesssim 0.35 \left( \frac{m_{\frac{3}{2}}}{40} \right)^2. \quad (5.17)$$

Notice that  $\delta m_e^2$  must necessarily be positive.

So, if we assume that the one-loop  $\beta_{\lambda_2}$  is dominated by its gauge contribution, consistent with our previous assumption that  $|q_\Phi g'| \gg |\lambda_{1,2,3}|$ , we obtain

$$\delta m_{l,e}^2 \simeq \frac{q_{L,E}(q_\Phi^2 + 2q_N^2)}{2q_\Phi} \frac{g'^2 m_{\frac{3}{2}}^2 \text{Tr } \lambda_2^2}{(16\pi^2)^2}. \quad (5.18)$$

Now  $q_\Phi = 4q_L + 2q_E$ , so we see that it is easy to obtain the correct sign for  $\delta m_e^2$ .

For  $q_L = 0$ , we find

$$\delta m_e^2 \simeq 3q_E^2 \frac{g'^2 m_{\frac{3}{2}}^2 \text{Tr } \lambda_2^2}{2(16\pi^2)^2} \quad (5.19)$$

or

$$1.6 \lesssim q_E^2 g'^2 \text{Tr } \lambda_2^2 \lesssim 3.6. \quad (5.20)$$

Of course with  $q_L = 0$ , we have  $\delta m_l^2 = 0$ ; but as describe earlier, it was shown in Ref. [10] that acceptable slepton masses nevertheless result when we run down to low energies. Clearly there are similar contributions to the masses of the other matter fields similar to Eq. (5.18), thus for example

$$\delta m_{h_1, h_2}^2 \simeq \frac{q_{H_1, H_2}(q_\Phi^2 + 2q_N^2)}{2q_\Phi} \frac{g'^2 m_{\frac{3}{2}}^2 \text{Tr } \lambda_2^2}{(16\pi^2)^2}. \quad (5.21)$$

In the notation of Ref. [10], Eq. (5.18), for example, is simply replaced by  $\delta m_l^2 = Lk'$  and  $\delta m_e^2 = ek'$  with  $(L, e)$  replacing  $q_{L,E}$ , and all results presented for  $k' = 1(\text{TeV})^2$ .

We emphasise once again the contrast between our model and conventional versions of the NMSSM, which does not, in basic form, contain an extra gauged  $U(1)$ , but where a vev (of the scale of supersymmetry breaking) for the gauge singlet  $s$  generates a Higgs  $\mu$ -term in much the same way, as is done here. However, while in the NMSSM case the  $s$  fields are very much part of the Higgs spectrum, here, in spite of the comparatively small  $s$ -vev, the  $s$ -quanta obtain large supersymmetric masses and are decoupled from the low energy physics, which becomes simply that of the MSSM. Another nice feature is the natural emergence of the see-saw mechanism via the spontaneous breaking of the  $U(1)'$ . Evidently it will be feasible to associate the  $U(1)'$  breaking scale given by Eq. (5.7) with the scale of gauge unification.

Although, as indicated above, we will be regarding  $M$  as source of significant physics, it is worth briefly considering the limit  $M \rightarrow \infty$ . In that limit, the theory becomes simply the MSSM (including the Higgs  $\mu_h$ -term) with the soft breaking terms given in Eq. (2.1)-Eq. (2.3) including the additional  $kY$  term, which resolves the tachyon problem. The explicit form of the terms proportional to the gravitino mass in these equations is easily derived using the conformal compensator field as described in Ref. [5]. Of course, although the resulting  $kY$  term in Eq. (2.3) has the *form* of an FI term, in the effective theory (for  $M \rightarrow \infty$ )  $U(1)'$  is not gauged and so we do not fall foul of the strictures of Ref. [8]. The conformal compensator field does not provide us with a straightforward derivation of Eq. (2.4); as described earlier, we will, like most previous authors, rely on the electroweak minimisation process to determine the Higgs  $B$ -term.

## 5.2 The $h_{1,2}, s$ extremum

We now consider the scalar potential

$$\begin{aligned}
V = & \lambda_3^2(|h_1 s|^2 + |h_2 s|^2) + |\lambda_3 h_1 h_2 - M^2|^2 + \frac{1}{2}g'^2 q_{H_1}^2 (|h_1|^2 - |h_2|^2)^2 \\
& + \frac{1}{8}g_1^2 (h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{1}{8}g_2^2 \sum_a (h_1^\dagger \sigma^a h_1 + h_2^\dagger \sigma^a h_2)^2 \\
& + m_{h_1}^2 |h_1|^2 + m_{h_2}^2 |h_2|^2 + m_s^2 |s|^2 + \rho M^2 m_{\frac{3}{2}} (s + s^*) \\
& + h_{\lambda_3} h_1 h_2 s + c.c..
\end{aligned} \tag{5.22}$$

In Eq. (5.22) we have written the  $U(1)_Y$  gauge coupling as  $g_1$ , although its normalisation corresponds to the usual SM convention, not that appropriate for  $SU(5)$  unification. This is to avoid confusion with the  $U(1)'$  coupling,  $g'$ .

We see that the potential is very similar to Eq. (5.1), the main difference being the presence of  $SU(2)$  and  $U(1)_Y$  D-terms. To leading order in  $M$ , only the  $SU(2)$  D-term

depends on the relative direction in  $SU(2)$ -space of the two doublets; it follows that we can choose without loss of generality to set  $h_1 = (v_1/\sqrt{2}, 0)$  and  $h_2 = (0, v_2/\sqrt{2})$ , as in electroweak breaking, in order to obtain zero for the  $SU(2)$  D-term for  $v_1 = v_2$ . Minimisation of the potential then proceeds in a similar way to the previous section (with the replacement  $\lambda_1 \rightarrow \lambda_3$ ) leading to

$$V_h = \frac{M^2}{\lambda_3} \left( m_{h_1}^2 + m_{h_2}^2 - \frac{h_{\lambda_3}^2}{2\lambda_3^2} \right) \quad (5.23)$$

at the extremum. Here

$$h_{\lambda_3} = -m_{\frac{3}{2}} \frac{\lambda_3}{16\pi^2} (\text{Tr } Y_E Y_E^\dagger + 3 \text{Tr } Y_D Y_D^\dagger + 3 \text{Tr } Y_U Y_U^\dagger + \lambda_1^2 + 4 \text{Tr } \lambda_3^2 - 3g_2^2 - g_1^2 - 4q_{H_1}^2 g'^2). \quad (5.24)$$

Let us compare the result for  $V_h$  with that obtained for  $V_\phi$ , in the previous section, Eq. (5.12). If we assume that the  $g'$  terms dominate throughout we obtain simply

$$V_\phi = -\frac{M^2}{\lambda_1} \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right)^2 [4Qq_\Phi^2 + 8q_\Phi^4] \quad (5.25)$$

and

$$V_h = -\frac{M^2}{\lambda_3} \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right)^2 [4Qq_{H_1}^2 + 8q_{H_1}^4], \quad (5.26)$$

where we have written the one loop  $g'$   $\beta$ -function as

$$\beta_{g'} = Q \frac{g'^3}{16\pi^2} \quad (5.27)$$

and

$$\begin{aligned} Q &= n_G \left( \frac{40}{3} q_L^2 + 8q_E^2 + 16q_E q_L \right) + 36q_L^2 + 40q_E q_L + 12q_E^2 \\ &= 76q_L^2 + 36q_E^2 + 88q_E q_L \quad (\text{for } n_G = 3). \end{aligned} \quad (5.28)$$

The coefficient  $Q$  is in general large, and larger than both  $q_\Phi^2$  and  $q_{H_1}^2$ , so the condition for the  $\phi, \bar{\phi}, s$  extremum to have a lower energy than the  $h_1, h_2, s$  one may be written

$$\lambda_1 \left( \frac{q_{H_1}}{q_\Phi} \right)^2 \lesssim \lambda_3. \quad (5.29)$$

Alternatively, for the specific choice  $q_L = 0$ , which we will see in the next section leads to an acceptable electro-weak vacuum, we find that the same condition becomes

$$\frac{19}{88} \lambda_1 \lesssim \lambda_3. \quad (5.30)$$

## 6. The sparticle spectrum

In this section we calculate sparticle spectra for the sAMSB model, and compare the results with typical mAMSB spectra. We shall be interested in seeking regions of parameter space with a “high” Higgs mass - that is, close to about 125 GeV as suggested by recent LHC data [1, 2] - and a supersymmetric contribution to the muon anomalous magnetic moment  $\delta a_\mu$  compatible with the experimental deviation from the Standard Model prediction,  $\delta a_\mu^{\text{exp}} = 29.5(8.8) \times 10^{-10}$  [25]. We will also wish to remain consistent with the negative results of recent LHC supersymmetry searches, see for example Refs. [26, 27].

We use the methodology of Ref. [10], which, as explained in Section 2, can also be applied to mAMSB by replacing the characteristic  $(L, e)$  FI-type terms of sAMSB by a universal mass term  $m_0^2$ .

We begin by choosing input values for  $m_{\frac{3}{2}}$ ,  $\tan\beta$ ,  $L$ ,  $e$  and  $\text{sign}\mu_h$  at the gauge unification scale  $M_X$ . Then we calculate the appropriate dimensionless coupling input values at the scale  $M_Z$  by an iterative procedure involving the sparticle spectrum, and the loop corrections to  $\alpha_{1\dots 3}$ ,  $m_t$ ,  $m_b$  and  $m_\tau$ , as described in Ref. [28]. We define gauge unification by the meeting point of  $\alpha_1$  and  $\alpha_2$ ; this scale, of around  $10^{16}\text{GeV}$ , we assume to be equal or close to the scale of  $U(1)'$  breaking. For the top quark pole mass we use  $m_t = 172.9\text{GeV}$ . All calculations are done in the approximation that we retain only third generation Yukawa couplings,  $\lambda_{t,b,\tau}$ ; thus the squarks and sleptons of the second generation are degenerate with the corresponding ones of the first generation.

We then determine a given sparticle pole mass by running the dimensionless couplings up to a certain scale chosen (by iteration) to be equal to the pole mass itself, and then implementing full one-loop corrections from Ref. [28], and two-loop corrections to the top quark mass [29]. We use two-loop anomalous dimensions and  $\beta$ -functions throughout.

### 6.1 Mass spectra in sAMSB

We display some examples of spectra in Tables 2-5. In each Table, the columns are for different gravitino masses, all with  $L = 0$  with  $e$  increasing with increasing gravitino mass so as to remain within the allowed  $(L, e)$  region; obviously  $e$  scales like  $m_{\frac{3}{2}}^2$  from Eq. (5.18). (As already indicated, we input  $(L, e)$  at  $M_X$ , so the allowed  $(L, e)$  region corresponds to that in Ref. [10] rather than that in Ref. [9]). In Tables 2,4 the  $(L, e)$  values are in the centre of the allowed  $(L, e)$  region (at least for smaller values of  $m_{\frac{3}{2}}$ ), whereas in Tables 3,5  $e$  is smaller so that lighter sleptons result. We see that  $\mu_h/m_{\frac{3}{2}}$  varies little with  $m_{\frac{3}{2}}$ ; for example in Table 2 changing from 0.014 at  $m_{\frac{3}{2}} = 40\text{TeV}$  to

0.012 at  $m_{\frac{3}{2}} = 140\text{TeV}$ . We thus find from Eq. (5.13) that

$$\lambda_1 \lambda_3 \frac{2q_\phi^2 g'^2}{\lambda_1^2} \simeq 2.2 \quad (6.1)$$

in order for the electro-weak vacuum to exist. We shall return to this formula when we have discussed the cosmological constraints.

In Tables 2, 3 we have  $\tan \beta = 10$ , whereas in Table 4,5 we have  $\tan \beta = 16$ . Increasing  $\tan \beta$  generally leads to a slight increase in the light Higgs mass  $m_h$ , and in the Table 5 case a much larger *decrease* in the heavy Higgs masses; this decrease is a signal of the fact that (for given  $m_{\frac{3}{2}}$ ,  $L$ ,  $e$ ) there is an upper limit on  $\tan \beta$ ; above that limit, the electroweak vacuum fails.

Increasing the scale of supersymmetry breaking (by increasing  $m_{\frac{3}{2}}$ ) will, generally speaking, allow us to remain compatible with the more stringent limits on BSM physics emerging from LHC searches and  $B$ -decay. Recent LHC publications on supersymmetry searches (see for example Refs. [26, 27]) tend to focus on sparticle spectra which are not compatible with AMSB; but it seems clear that for  $m_{\frac{3}{2}} \gtrsim 60\text{TeV}$  or so, our model is not (yet) ruled out. One search result that explicitly targets anomaly mediation is that of Ref. [30]; this sets a lower limit on the wino mass of  $92\text{GeV}$ , which in sAMSB would correspond to  $m_{\frac{3}{2}} \simeq 28\text{TeV}$ .

Increasing  $m_{\frac{3}{2}}$  so as to reduce squark/gluino production will, however, reduce the supersymmetric contribution to the muon anomalous magnetic moment  $a_\mu$ , and hence the opportunity to account for the existing discrepancy between theory and experiment. But it is a feature of AMSB, and in particular sAMSB, that the sleptons are comparatively light compared to the gluino and squarks. Therefore it turns out to be possible to combine heavier coloured states with sleptons and electro-weak gauginos still light enough to contribute appreciably to  $a_\mu$ . We demonstrate this by including in the tables the result for the supersymmetric contribution to  $a_\mu$ . For  $m_{\frac{3}{2}} = 60\text{TeV}$ , the result is manifestly compatible with the afore-mentioned discrepancy.<sup>2</sup>

Notice that increasing  $m_{\frac{3}{2}}$  so as to increase  $m_h$  to bring it closer to the recent announcement of evidence [1, 2] for a SM-like Higgs in the region of  $125\text{GeV}$  can be done, but at the cost of reducing  $\delta a_\mu$ ; see the last column in Tables 4, 5. It also increases the degree of fine-tuning, as we shall discuss presently.

We can also increase  $\delta a_\mu$  by choosing  $(L, e)$  closer to one of the boundaries of the allowed region corresponding to either the charged slepton doublets or singlets becoming too light; but the effect of doing this is limited in that the gaugino masses are not sensitive to  $(L, e)$ . The bottom line is that with  $\tan \beta = 10$ , to account for the

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<sup>2</sup>We use the one-loop formulae of Ref. [31]; for a review and more references see Ref. [32].



$m_{\frac{3}{2}}$	40TeV	60TeV	80TeV	100TeV	120TeV	140TeV
$(L, e)$	(0, 0.25)	(0, 0.5625)	(0, 1)	(0, 1.5625)	(0, 2.25)	(0, 3.0625)
$\tilde{g}$	900	1297	1684	2062	2434	2802
$\tilde{t}_1$	757	1054	1346	1633	1915	2120
$\tilde{t}_2$	507	723	925	1115	1298	1473
$\tilde{u}_L$	819	1181	1531	1875	2211	2542
$\tilde{u}_R$	766	1093	1408	1714	2012	2304
$\tilde{b}_1$	714	1023	1322	1614	1900	2181
$\tilde{b}_2$	946	1376	1798	2213	2624	3031
$\tilde{d}_L$	822	1183	1533	1876	2212	2544
$\tilde{d}_R$	955	1390	1816	2236	2651	3062
$\tilde{\tau}_1$	199	309	419	532	645	758
$\tilde{\tau}_2$	266	388	512	635	759	882
$\tilde{e}_L$	212	321	433	546	661	776
$\tilde{e}_R$	261	387	512	637	762	887
$\tilde{\nu}_e$	249	378	506	632	758	883
$\tilde{\nu}_\tau$	247	375	502	627	752	876
$\chi_1$	131	198	265	331	396	461
$\chi_2$	362	548	734	920	1107	1294
$\chi_3$	588	841	1084	1319	1549	1773
$\chi_4$	599	850	1091	1325	1552	1778
$\chi_1^\pm$	131	199	265	331	396	461
$\chi_2^\pm$	597	848	1089	1324	1552	1777
$h$	115	118	120	122	123	124
$H, A$	366	492	595	680	749	802
$H^\pm$	374	499	601	685	753	806
$\chi_1^\pm - \chi_1$ (MeV)	236	218	214	210	204	194
$\mu_h$	571	812	1041	1259	1470	1675
$\delta a_\mu$	$62 \times 10^{-10}$	$26 \times 10^{-10}$	$13 \times 10^{-10}$	$7.5 \times 10^{-10}$	$4.6 \times 10^{-10}$	$3.0 \times 10^{-10}$

**Table 2:** sAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_t = 172.9\text{GeV}$  and  $\tan \beta = 10$

whole of  $\delta a_\mu^{\text{exp}}$  we need a light higgs mass of around  $115 - 120\text{GeV}$ . Increasing  $\tan \beta$  also leads to larger  $\delta a_\mu$ , but also a *smaller* charged Higgs mass, and a potentially over-large contribution to the branching ratio  $B \rightarrow X_s \gamma$ . This effect is particularly noticeable in Table 4, where the heavy Higgs masses actually *decrease* as  $m_{\frac{3}{2}}$  is increased. We will return to this issue in Section 6.3.

As in most versions of AMSB, the LSP is mostly neutral wino, with the charged wino a few hundred MeV heavier.

$m_{\frac{3}{2}}$	40TeV	60TeV	80TeV	100TeV	120TeV	140TeV
$(L, e)$	(0, 0.16)	(0, 0.36)	(0, 0.64)	(0, 1)	(0, 1.44)	(0, 1.96)
$\tilde{g}$	900	1297	1684	2063	2435	2802
$\tilde{t}_1$	770	1071	1369	1662	1951	2237
$\tilde{t}_2$	548	792	1023	1245	1460	1668
$\tilde{u}_L$	825	1191	1545	1892	2233	2568
$\tilde{u}_R$	795	1141	1474	1798	2116	2428
$\tilde{b}_1$	723	1037	1342	1640	1933	2891
$\tilde{b}_2$	909	1320	1721	2116	2506	2237
$\tilde{d}_L$	829	1194	1547	1894	2234	2922
$\tilde{d}_R$	919	1334	1740	2140	2532	2569
$\tilde{\tau}_1$	119	194	270	346	424	502
$\tilde{\tau}_2$	198	281	366	452	537	623
$\tilde{e}_L$	145	219	295	373	452	532
$\tilde{e}_R$	187	275	363	451	539	627
$\tilde{\nu}_e$	170	263	354	444	533	622
$\tilde{\nu}_\tau$	167	259	349	437	525	612
$\chi_1$	131	198	265	330	395	460
$\chi_2$	363	549	736	922	1109	1296
$\chi_3$	635	916	1186	1450	1709	1964
$\chi_4$	645	922	1192	1455	1713	1968
$\chi_1^\pm$	131	199	265	330	395	460
$\chi_2^\pm$	643	921	1190	1454	1712	1967
$h$	115	118	120	122	123	124
$H, A$	499	710	907	1094	1274	1448
$H^\pm$	506	716	911	1098	1277	1451
$\chi_1^\pm - \chi_1$ (MeV)	223	213	212	210	204	197
$\mu_h$	618	886	1142	1390	1470	1867
$\delta a_\mu$	$62 \times 10^{-10}$	$27 \times 10^{-10}$	$15 \times 10^{-10}$	$9.2 \times 10^{-10}$	$6.2 \times 10^{-10}$	$4.5 \times 10^{-10}$

**Table 3:** sAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_t = 172.9\text{GeV}$  and  $\tan\beta = 10$

## 6.2 Comparison with mAMSB

It is interesting to compare the sAMSB spectra presented in Tables 2-5 with some mAMSB spectra.

In Table 6 we present results for  $m_{\frac{3}{2}} = 60\text{TeV}$ , for different values of  $m_0$ . The second column of this table corresponds to the Benchmark Point mAMSB1.3 of Ref. [33]; our results for the masses agree reasonably well with those presented there: for example, the gluino masses differ by 2%, and the lightest third generation squarks by 1%. They are also not inconsistent with those of Ref. [34], who quote an upper limit for  $m_h$  of  $120.4\text{GeV}$ ; note that there the parameter scan is restricted to  $m_0 < 2\text{TeV}$ . For a

$m_{\frac{3}{2}}$	40TeV	60TeV	80TeV	100TeV	120TeV	140TeV
$(L, e)$	(0, 0.25)	(0, 0.5625)	(0, 1)	(0, 1.5625)	(0, 2.25)	(0, 3.0625)
$\tilde{g}$	899	1297	1683	2062	2434	2801
$\tilde{t}_1$	750	1041	1328	1612	1892	2168
$\tilde{t}_2$	504	721	924	1116	1300	1745
$\tilde{u}_L$	819	1181	1532	1875	2211	2543
$\tilde{u}_R$	766	1094	1409	1714	2012	2305
$\tilde{b}_1$	703	1007	1301	1590	1873	2153
$\tilde{b}_2$	929	1352	1768	2177	2582	2983
$\tilde{d}_L$	823	1183	1534	1876	2213	2544
$\tilde{d}_R$	955	1391	1812	2236	2651	3062
$\tilde{\tau}_1$	182	291	400	511	621	733
$\tilde{\tau}_2$	271	391	512	633	755	877
$\tilde{e}_L$	212	321	433	546	660	776
$\tilde{e}_R$	262	387	512	638	762	887
$\tilde{\nu}_e$	249	378	506	632	758	883
$\tilde{\nu}_\tau$	244	372	497	621	752	867
$\chi_1$	132	199	265	331	396	461
$\chi_2$	362	548	734	920	1107	1294
$\chi_3$	585	836	1077	1311	1539	1763
$\chi_4$	594	843	1083	1316	1544	1767
$\chi_1^\pm$	132	199	265	331	396	461
$\chi_2^\pm$	592	842	1082	1315	1543	1766
$h$	116	119	121	123	124	125
$H, A$	284	366	417	440	430	374
$H^\pm$	285	375	425	447	438	384
$\chi_1^\pm - \chi_1$ (MeV)	229	216	213	210	204	194
$\mu_h$	566	806	1032	1249	1458	1662
$\delta a_\mu$	$1 \times 10^{-8}$	$41 \times 10^{-10}$	$21 \times 10^{-10}$	$12 \times 10^{-10}$	$7.5 \times 10^{-10}$	$4.9 \times 10^{-10}$

**Table 4:** sAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_t = 172.9\text{GeV}$  and  $\tan\beta = 16$

detailed comparison of mAMSB results with recent LHC data see Ref. [35]. We see that by increasing  $m_0$ , we can eventually make all the squarks heavier than the gluino; this is not possible in sAMSB, because increasing  $L$  and  $e$  soon leads to loss of the electro-weak vacuum. We will discuss this fact in more detail in Section 6.3.

In Table 7 we present the corresponding results for  $m_{\frac{3}{2}} = 140\text{TeV}$ . Note the (comparitively) light sleptons in column 2 of this Table; these occur because for these values the  $m_0^2$  contribution to the slepton (masses)<sup>2</sup> almost cancels the (negative)  $m_{\frac{3}{2}}^2$  one. (We do not give results in Table 7 for  $m_0 = 450\text{GeV}$ , because in that case there are still tachyonic sleptons). This is analogous to being close to a boundary in the allowed

$m_{\frac{3}{2}}$	40TeV	60TeV	80TeV	100TeV	120TeV	140TeV
$(L, e)$	(0, 0.18)	(0, 0.405)	(0, 0.72)	(0, 1.125)	(0, 1.62)	(0, 1.96)
$\tilde{g}$	899	1297	1684	2062	2434	2801
$\tilde{t}_1$	761	1056	1348	1635	1918	2199
$\tilde{t}_2$	536	775	1001	1218	1426	1629
$\tilde{u}_L$	928	1191	1543	1889	2228	2563
$\tilde{u}_R$	824	1131	1460	1780	2094	2402
$\tilde{b}_1$	710	1019	1348	1611	1898	2181
$\tilde{b}_2$	901	1309	1708	2101	2488	2872
$\tilde{d}_L$	828	1192	1545	1890	2230	2564
$\tilde{d}_R$	928	1347	1757	2161	2559	2954
$\tilde{\tau}_1$	111	196	280	364	448	532
$\tilde{\tau}_2$	223	314	405	498	590	683
$\tilde{e}_L$	163	245	331	418	508	595
$\tilde{e}_R$	206	304	401	499	596	693
$\tilde{\nu}_e$	190	293	393	492	591	689
$\tilde{\nu}_\tau$	184	284	381	477	573	668
$\chi_1$	132	199	265	330	396	460
$\chi_2$	363	549	735	922	1108	1295
$\chi_3$	621	892	1156	1412	1664	1910
$\chi_4$	630	898	1161	1417	1668	1914
$\chi_1^\pm$	132	199	265	331	396	460
$\chi_2^\pm$	628	898	1160	1416	1667	1913
$h$	116	119	121	123	124	125
$H, A$	410	577	729	869	1001	1125
$H^\pm$	419	583	734	873	1005	1129
$\chi_1^\pm - \chi_1$ (MeV)	218	212	211	209	204	195
$\mu_h$	603	863	1111	1379	1584	1852
$\delta a_\mu$	$101 \times 10^{-10}$	$44 \times 10^{-10}$	$24 \times 10^{-10}$	$14 \times 10^{-10}$	$9.5 \times 10^{-10}$	$6.5 \times 10^{-10}$

**Table 5:** sAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_t = 172.9\text{GeV}$  and  $\tan\beta = 16$

$(L, e)$  space in the sAMSB case, and, as there, does not in itself result in a large  $\delta a_\mu$ , because the wino masses are unaffected. Moreover, away from the  $(L, e)$  boundary (in sAMSB) the slepton masses remain relatively small, whereas for fixed  $m_{\frac{3}{2}}$ , increasing  $m_0$  (in mAMSB) leads rapidly to larger slepton masses.

It is interesting that in mAMSB, increasing  $m_0$  (for fixed  $m_{\frac{3}{2}}$ ) leads to a slight decrease in  $\mu_h$ , and a consequent slight decrease in the masses of the heavy neutralinos and chargino. Note also that the supersymmetric contribution to  $a_\mu$  is compatible with  $\delta a_\mu^{\text{exp}}$  for  $m_0 = 450\text{GeV}$ , in Table 6, but decreases rapidly as  $m_0$  increases. If we increase  $m_{\frac{3}{2}}$  to 140TeV as in Table 7, we are able to obtain  $m_h = 125\text{GeV}$ , but, as in sAMSB

$m_0$	450	900	1800	2700
$\tilde{g}$	1310	1342	1398	1438
$\tilde{t}_1$	1156	1303	1783	2398
$\tilde{t}_2$	940	1052	1384	1804
$\tilde{u}_L$	1295	1499	2135	2912
$\tilde{u}_R$	1285	1489	2126	2903
$\tilde{b}_1$	1120	1278	1773	2392
$\tilde{b}_2$	1288	1489	2121	2892
$\tilde{d}_L$	1287	1491	2128	2904
$\tilde{d}_R$	1303	1506	2141	2917
$\tilde{\tau}_1$	355	851	1764	2664
$\tilde{\tau}_2$	399	870	1774	2671
$\tilde{e}_L$	381	865	1778	2680
$\tilde{e}_R$	390	871	1784	2687
$\tilde{\nu}_e$	372	861	1776	2679
$\tilde{\nu}_\tau$	367	856	1768	2668
$\chi_1$	199	200	201	202
$\chi_2$	550	555	558	559
$\chi_3$	1031	1027	1004	950
$\chi_4$	1037	1032	1009	956
$\chi_1^\pm$	200	201	201	202
$\chi_2^\pm$	1036	1031	1009	955
$h$	118	119	120	122
$H, A$	1076	1314	2006	2802
$H^\pm$	1079	1317	2008	2804
$\chi_1^\pm - \chi_1$ (MeV)	209	209	208	209
$\mu_h$	1000	989	956	889
$\delta a_\mu$	$22 \times 10^{-10}$	$6.1 \times 10^{-10}$	$0.57 \times 10^{-10}$	$0.10 \times 10^{-10}$

**Table 6:** mAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_{\frac{3}{2}} = 60\text{TeV}$ ,  $m_t = 172.9\text{GeV}$  and  $\tan\beta = 10$

at the price of a small contribution to  $\delta a_\mu$ .

### 6.3 Fine tuning

Noting that as  $m_{\frac{3}{2}}$  is increased we find that  $\mu_h$  increases, we should comment on the issue of the fine-tuning required to produce the electro-weak scale. From the well-known tree level relation

$$\frac{m_{h_1}^2 - m_{h_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu_h^2 = \frac{1}{2} M_Z^2 \quad (6.2)$$

$m_0$	900	1800	2700
$\tilde{g}$	2824	2881	2939
$\tilde{t}_1$	2382	2682	3114
$\tilde{t}_2$	2038	2248	2548
$\tilde{u}_L$	2776	3162	3720
$\tilde{u}_R$	2756	3143	3703
$\tilde{b}_1$	2362	2668	3105
$\tilde{b}_2$	2704	3085	3636
$\tilde{d}_L$	2757	3144	3707
$\tilde{d}_R$	2795	3179	3735
$\tilde{\tau}_1$	620	1652	2573
$\tilde{\tau}_2$	710	1691	2605
$\tilde{e}_L$	707	1717	2634
$\tilde{e}_R$	723	1707	2643
$\tilde{\nu}_e$	703	1705	2632
$\tilde{\nu}_\tau$	670	1678	2560
$\chi_1$	461	464	465
$\chi_2$	1297	1306	1311
$\chi_3$	2240	2211	2162
$\chi_4$	2243	2214	2164
$\chi_1^\pm$	461	464	465
$\chi_2^\pm$	2242	2213	2164
$h$	125	126	126
$H, A$	2186	2618	3214
$H^\pm$	2188	2620	3216
$\chi_1^\pm - \chi_1$ (MeV)	191	175	161
$\mu_h$	2136	2095	2032
$\delta a_\mu$	$5.8 \times 10^{-10}$	$0.95 \times 10^{-10}$	$0.22 \times 10^{-10}$

**Table 7:** mAMSB mass spectra (in GeV), and  $\delta a_\mu$  for  $m_{\frac{3}{2}} = 140\text{TeV}$ ,  $m_t = 172.9\text{GeV}$  and  $\tan\beta = 16$

we see that unless  $|m_{h_1}^2| \gg |m_{h_2}^2|$  then, for typical values of  $\tan\beta$ , we have

$$\mu_h^2 \simeq -m_{h_2}^2 - \frac{1}{2}M_Z^2 \quad (6.3)$$

which since generically  $|m_{h_2}| \gg M_Z$  represents a fine tuning, sometimes called the “little hierarchy” problem.

One might have hoped, since  $q_{H_2} = q_L + q_E > 0$ , to reduce  $|m_{h_2}^2|$ , and hence  $\mu_h^2$ , by increasing  $q_L + q_E$ ; see Eq. (5.21). But from Fig. 1 of Ref. [10] we see  $L + e$  is severely constrained by the requirement of a stable electroweak vacuum; the failure of this is

manifested by a tachyonic  $m_A$ . The tree formula for  $m_A$  is

$$m_A^2 = 2\mu_h^2 + m_{h_1}^2 + m_{h_2}^2 \simeq m_{h_1}^2 - m_{h_2}^2 - M_Z^2 \quad (6.4)$$

and it is apparent from Eq. (5.21) that the overall effect of increasing  $q_L + q_E$  actually decreases  $m_A^2$ .

For example, if we use  $m_{\frac{3}{2}} = 80\text{TeV}$  and  $(L, e) = (0, 1.2)$ , then we find that  $m_A$  is sharply reduced to  $295\text{GeV}$  while  $\mu_h$  changes only to  $980\text{GeV}$ . A small further increase in  $e$  takes  $m_A$  rapidly to zero. A similar outcome is the result of increasing  $\tan\beta$ . For example, with  $m_{\frac{3}{2}} = 80\text{TeV}$  and  $(L, e) = (0, 1)$ , as in the fourth column of Table 2,  $\mu_h$  decreases with increasing  $\tan\beta$  but  $m_A$  decreases more sharply. For  $\tan\beta = 19$ , we find  $\mu_h = 1025\text{GeV}$ , but  $m_A = 207\text{GeV}$ , and for  $\tan\beta = 20$ ,  $m_A^2 < 0$ .

If we increase  $m_{\frac{3}{2}}$  then the upper limit on  $\tan\beta$  decreases; for example with  $m_{\frac{3}{2}} = 120\text{TeV}$  and  $(L, e) = (0, 2.25)$  as in the sixth column of Table 2, we find that the maximum value of  $\tan\beta$  is  $\tan\beta = 17$ , with  $m_h = 124\text{GeV}$  and  $m_A = 309\text{GeV}$ , and  $\delta a_\mu = 7.9 \times 10^{-10}$ . Note that  $\delta a_\mu$  increases as  $\tan\beta$  increases; however, in Table 4, the concomitant decrease in the Higgs masses (in particular the charged Higgs mass) leads to an increased supersymmetric contribution to the branching ratio for  $B \rightarrow X_s \gamma$ , and potential conflict with experiment. See Figure 4 of Ref. [36]. This problem is avoided in Table 5; but with  $m_{\frac{3}{2}}$  large enough to produce  $m_h = 125\text{GeV}$ , there is no region in  $(L, e)$  space permitting a  $\tan\beta$  large enough to generate  $\delta a_\mu \approx 20 - 30 \times 10^{-10}$ .

Within the context of our model we see no clean way to avoid the fine-tuning problem. It is interesting to note that with the alternative GUT-compatible assignment considered in Section 5 of Ref. [10],  $L + e$  can be increased if desired (see Fig. 2 of that reference). However in that case we have  $q_{H_1} = -e - L$  and  $q_{H_2} = -2e$ , so increasing  $L + e$  does not reduce  $|m_2^2|$  or  $m_A^2$ .

## 7. Cosmological history

### 7.1 F-term inflation

As detailed in a previous paper [7], the theory naturally produces F-term inflation [37]-[39], with the singlet scalar  $s$  as the inflaton. In this paper we are assuming that the FI-term vanishes, which considerably simplifies the radiative corrections driving the evolution of  $s$  during inflation. We also assume that the quartic term in  $s$  in the Kähler potential is negligible.





At large  $s$  and vanishing  $\phi$ ,  $\bar{\phi}$ ,  $h_1$  and  $h_2$ , and neglecting soft terms, we have

$$V_{\text{tree}} = M^4 + \Delta V_1, \quad (7.2)$$

where  $\Delta V_1$  represents the one-loop corrections, given as usual by

$$\Delta V_1 = \frac{1}{64\pi^2} \text{Str} [(M^2(s))^2 \ln(M^2(s)/\mu^2)]. \quad (7.3)$$

Here

$$\text{Str} \equiv \sum_{\text{scalars}} -2 \sum_{\text{fermions}} +3 \sum_{\text{vectors}}. \quad (7.4)$$

In the absence of the FI term,  $\Delta V$  is in fact dominated by the  $\Phi$ ,  $\bar{\Phi}$  and  $H_{1,2}$  subsystems, and the contribution to the one-loop scalar potential is [7]

$$\begin{aligned} \Delta V_1 = & \frac{1}{32\pi^2} \left[ (\lambda_1^2 s^2 + \lambda_1 M^2)^2 \ln \left( \frac{\lambda_1^2 s^2 + \lambda_1 M^2}{\mu^2} \right) + (\lambda_1^2 s^2 - \lambda_1 M^2)^2 \ln \left( \frac{\lambda_1^2 s^2 - \lambda_1 M^2}{\mu^2} \right) \right. \\ & + 2(\lambda_3^2 s^2 + \lambda_3 M^2)^2 \ln \left( \frac{\lambda_3^2 s^2 + \lambda_3 M^2}{\mu^2} \right) + 2(\lambda_3^2 s^2 - \lambda_3 M^2)^2 \ln \left( \frac{\lambda_3^2 s^2 - \lambda_3 M^2}{\mu^2} \right) \\ & \left. - 2\lambda_1^4 s^4 \ln \left( \frac{\lambda_1^2 s^2}{\mu^2} \right) - 4\lambda_3^4 s^4 \ln \left( \frac{\lambda_3^2 s^2}{\mu^2} \right) \right]. \end{aligned} \quad (7.5)$$

For values of  $s$  for which  $\lambda_{1,3}s^2 \gg M^2$  it is easy to show that, after removing a finite local counterterm, this reduces to

$$V(s) \simeq M^4 \left[ 1 + \alpha \ln \frac{2s^2}{s_c^2} \right], \quad (7.6)$$

where

$$\alpha = \frac{\lambda^2}{16\pi^2}, \quad \lambda = \sqrt{\lambda_1^2 + 2\lambda_3^2}, \quad s_c^2 = M^2/\lambda. \quad (7.7)$$

Note that neglecting the linear soft term  $\rho M^2 m_{\frac{3}{2}} s + \text{c.c.}$  is equivalent to assuming

$$\rho \ll \frac{\lambda^3}{16\pi^2} \frac{s_c}{m_{\frac{3}{2}}}. \quad (7.8)$$

With the parameterisation (7.6), the scalar and tensor power spectra  $\mathcal{P}_s$ ,  $\mathcal{P}_t$  and the scalar spectral index  $n_s$  generated  $N$  e-foldings before the end of inflation are

$$\mathcal{P}_s(k) \simeq \frac{1}{24\pi^2} \frac{2N_k}{\alpha} \left( \frac{M}{m_p} \right)^4 = \frac{4N_k}{3} \left( \frac{s_c}{m_p} \right)^4, \quad (7.9)$$

$$\mathcal{P}_t(k) \simeq \frac{1}{6\pi^2} \left( \frac{M}{m_p} \right)^4 = \frac{8}{3} \left( \frac{s_c}{m_p} \right)^4, \quad (7.10)$$

$$n_s \simeq \left( 1 - \frac{1}{N_k} \right). \quad (7.11)$$

The WMAP7 best-fit values for  $\mathcal{P}_s(k_0)$  and  $n_s$  at  $k = k_0 = 0.002 \text{ hMpc}^{-1}$  in the standard  $\Lambda\text{CDM}$  model are [40]

$$\mathcal{P}_s(k_0) = (2.43 \pm 0.11) \times 10^{-9}, \quad n_s = 0.963 \pm 0.012(68\%\text{CL}), \quad (7.12)$$

which correspond to

$$\frac{s_c}{m_p} \simeq 2.9 \times 10^{-3} \left( \frac{27}{N_{k_0}} \right)^{\frac{1}{4}}, \quad N_{k_0} = 27_{-7}^{+13}. \quad (7.13)$$

There is an approximately  $2\sigma$  discrepancy with the standard Hot Big Bang result  $N_{k_0} \simeq 55 + \ln(T_{\text{rh}}/10^{15} \text{ GeV})$ . We will see later how this is ameliorated by  $N_\theta \simeq 17$  e-foldings of thermal inflation, reducing the discrepancy to approximately  $1\sigma$ .

If  $\lambda_3 > \lambda_1$ , inflation ends at the critical value  $s_{c1}^2 = M^2/\lambda_1$ , followed by transition to the  $U(1)'$ -broken phase described by Eq. (5.2)-Eq. (5.4). On the other hand, if  $\lambda_3 < \lambda_1$ , we find that  $s_{c3}^2 = M^2/\lambda_3$ , and the Higgses develop vevs of order the unification scale rather than  $\phi, \bar{\phi}$ .

At first sight this rules out this latter possibility and in [7] we did not explore it. However, we saw in Section 5 that the condition for the correct (small Higgs vev) electroweak vacuum to have the lowest energy density (5.29) is slightly less restrictive than the condition for inflation to exit to the  $\phi\text{-}\bar{\phi}$  direction, and that there is a range of parameters

$$\lambda_1 \left( \frac{q_{H_1}}{q_\Phi} \right)^2 \lesssim \lambda_3 < \lambda_1 \quad (7.14)$$

for which the universe exits to the false high Higgs vev  $h$ -vacuum. It then should evolve to the true ground state: in this section we will see that this evolution leads to a very interesting cosmological history, with some distinctive features.

## 7.2 Reheating

If inflation exits to the  $h$ -vacuum the symmetry-breaking is

$$SU(2) \times U(1)_Y \times U(1)' \rightarrow U(1)_{\text{em}} \times U(1)'', \quad (7.15)$$

where the  $U(1)''$  is generated by the linear combination of hypercharge and  $U(1)'$  generators which leaves the Higgses invariant:

$$Y'' = Y - \frac{Y'}{q_L + q_E}. \quad (7.16)$$

Topologically, the symmetry-breaking is the same as in the Standard Model, and hence cosmic strings are not formed at this transition.

Reheating after hybrid inflation [41] is expected in our model to be very rapid, as the non-perturbative field interactions of the scalars with fermions [42] and with gauge fields [43] are very efficient at transferring energy out of the zero-momentum modes of the fields  $s$ ,  $h_1$  and  $h_2$ . Higgs modes decay rapidly into  $b$  quarks, leading to the universe regaining a relativistic equation of state in much less than a Hubble time. Hence the universe thermalises at a temperature  $T_{\text{rh}} \simeq M$ .

One notices that before thermal effects and soft terms are taken into account, the minimum of the scalar potential is determined by the requirement that both the F- and D-terms vanish. The vanishing of the D-terms ensures that  $|\phi| = |\bar{\phi}|$ ,  $|h_1| = |h_2|$  and  $h_1^\dagger h_2 = 0$ , while the vanishing of the F-term is assured by  $\lambda_1 \phi \bar{\phi} - \lambda_3 h_1 h_2 = M^2$ . The minimum can therefore be parametrized by an SU(2) gauge transformation and angles  $\chi, \varphi$  defined by

$$\begin{aligned} \langle h_1 \rangle &\simeq -i\sigma_2 \langle h_2 \rangle^* \simeq \left( \frac{M}{\sqrt{\lambda_3}} \cos \chi, 0 \right), \\ \langle \phi \rangle &\simeq \langle \bar{\phi}^* \rangle \simeq \frac{M}{\sqrt{\lambda_1}} \sin \chi e^{i\varphi}. \end{aligned} \quad (7.17)$$

The  $\varphi$  angle can always be removed by a  $U(1)''$  gauge transformation, so the physical flat direction just maps out the interval  $0 \leq \chi \leq \pi/2$ . At the special point  $\chi = 0$  the  $U(1)''$  symmetry is restored, and at  $\chi = \pi/2$  the  $SU(2) \otimes U(1)_Y$  is restored. Away from these special points only  $U(1)_{\text{em}}$  is unbroken.

With this parametrisation, it is straightforward to show that the leading  $O(M^2 m_{\frac{3}{2}}^2)$  terms in the effective potential for  $\chi$  are, after solving for  $s$ ,

$$V(\chi) \simeq -\frac{M^2}{2} \frac{\left( \tilde{h}_{\lambda_1} \sin^2 \chi + \tilde{h}_{\lambda_3} \cos^2 \chi \right)^2}{\lambda_1 \sin^2 \chi + \lambda_3 \cos^2 \chi} + M^2 \left( \frac{\bar{m}_\phi^2}{\lambda_1} \sin^2 \chi + \frac{\bar{m}_h^2}{\lambda_3} \cos^2 \chi \right), \quad (7.18)$$

where we have defined  $\tilde{h}_{\lambda_1} = \frac{h_{\lambda_1}}{\lambda_1}$ ,  $\tilde{h}_{\lambda_3} = \frac{h_{\lambda_3}}{\lambda_3}$ ,  $\bar{m}_\phi^2 = m_\phi^2 + m_{\bar{\phi}}^2$  and  $\bar{m}_h^2 = m_{h_1}^2 + m_{h_2}^2$ . A little more algebra demonstrates that

$$V''(0) \simeq \frac{2M^2}{\lambda_3} \left[ -\frac{\tilde{h}_{\lambda_3}^2}{2} \left( 2 \frac{\tilde{h}_{\lambda_1}}{\tilde{h}_{\lambda_3}} - \frac{\lambda_1}{\lambda_3} - 1 \right) + \bar{m}_\phi^2 \frac{\lambda_3}{\lambda_1} - \bar{m}_h^2 \right], \quad (7.19)$$

while the expansion around the true vacuum (the  $\phi$ -vacuum) at  $\chi = \pi/2$  is easily obtained by the replacements  $1 \leftrightarrow 3$  and  $\bar{m}_\phi^2 \leftrightarrow \bar{m}_h^2$ .

In sAMSB we have, under our assumption that the  $U(1)'$  couplings dominate the  $\beta$ -functions,

$$\tilde{h}_{\lambda_1} \simeq \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right) 4q_\phi^2, \quad \tilde{h}_{\lambda_3} \simeq \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right) 4q_{H_1}^2, \quad (7.20)$$

and

$$\bar{m}_h^2 \simeq - \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right)^2 4Qq_{H_1}^2, \quad \bar{m}_\phi^2 \simeq - \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right)^2 4Qq_\phi^2. \quad (7.21)$$

As pointed out in Section 5,  $Q$  is in general much larger than both  $q_\phi^2$  and  $q_{H_1}^2$ , so we see that the  $h$ -vacuum is unstable only if

$$\bar{m}_\phi^2 \frac{\lambda_3}{\lambda_1} - \bar{m}_h^2 \lesssim 0, \quad (7.22)$$

or

$$\frac{\lambda_3}{\lambda_1} \gtrsim \frac{q_{H_1}^2}{q_\phi^2}. \quad (7.23)$$

This coincides with the condition (5.29) that the  $h$ -vacuum has higher energy than the  $\phi$ -vacuum, and that the  $\phi$ -vacuum is stable.

Note that we can define a canonically normalised  $U(1)''$ -charged complex scalar modulus field  $X$ , related to  $\chi$  and  $\varphi$  in the neighbourhood of the  $h$ -vacuum by

$$X \simeq \sqrt{\frac{2M^2}{\lambda_1}} \chi e^{i\varphi} \quad (7.24)$$

and whose mass  $m_X$  is given by

$$m_X^2 \simeq \bar{m}_\phi^2 \frac{\lambda_3}{\lambda_1} - \bar{m}_h^2. \quad (7.25)$$

### 7.3 High temperature ground state

As we outlined in the previous section, reheating is expected to take place in much less than a Hubble time  $H \sim M^2/m_P$ , while the relaxation rate to the true ground state, the  $\phi$  vacuum, is from Eq. (7.25)  $m_X$ . Given that we expect  $m_X \sim 1$  TeV and  $M \sim 10^{14}$  GeV, reheating happens much faster than the relaxation, and the universe is trapped in the  $U(1)''$ -symmetric vacuum with the large Higgs vev.

The high temperature effective potential, or free energy density, can be written

$$f(X, T) = -\frac{\pi^2}{90} g_{\text{eff}}(X, T) T^4, \quad (7.26)$$

where  $g_{\text{eff}}(X, T)$  is the effective number of relativistic degrees of freedom at temperature  $T$ . At weak coupling,  $g_{\text{eff}}(X, T)$  can be calculated in the high-temperature expansion for all particles of mass  $m_i \ll T$  [44],

$$g_{\text{eff}}(X, T) \simeq g_{\text{eff}}^0 - \frac{90}{\pi^2} \sum_i c_{1,i} \frac{m_i^2(X)}{T^2}, \quad (7.27)$$

where  $g_{\text{eff}}^0$  is the effective number of degrees of freedom at  $X = 0$ , and  $c_1 = \frac{1}{24}, \frac{1}{48}$  for bosons and fermions respectively. For particles with  $m > T$ ,  $g_{\text{eff}}$  is exponentially suppressed.

We can see that  $X = 0$  is a local minimum for temperatures  $m_X \lesssim T \lesssim M$ , because away from that point the  $U(1)''$  gauge boson develops a mass  $q_\phi g' |X|$ , and so  $g_{\text{eff}}$  decreases. For similar reasons the  $\phi$ -vacuum at  $X_\phi \sim \sqrt{M^2/\lambda_1}$  is also a local minimum: away from that point the MSSM particles develop masses and again reduce  $g_{\text{eff}}$ .

In fact, by counting relativistic degrees of freedom at temperatures  $m_{\frac{3}{2}} \ll T \lesssim M$  one finds that  $X_\phi$  is the global minimum. In the  $h$ -vacuum the relativistic species are the  $\Phi, \bar{\Phi}$  chiral multiplets and the  $U(1)''$  gauge multiplet. In the  $\phi$ -vacuum, the particles of the MSSM are all light relative to  $T$ . Hence

$$f(0, T) \simeq -\frac{15}{2} \frac{\pi^2}{90} T^4, \quad (7.28)$$

$$f(X_\phi, T) \simeq -\frac{915}{4} \frac{\pi^2}{90} T^4. \quad (7.29)$$

The minima of the free energy density are separated by a free energy barrier of height  $\sim T^4$ . The transition rate can be calculated in the standard way [45] by calculating the free energy of the critical bubble  $E_c$ , and it is not hard to show that the transition rate is suppressed by a factor  $\exp(-X_\phi/T)$ . Hence we expect that the universe is trapped in the  $h$ -vacuum at temperatures  $T \gtrsim m_X/q_\phi g'$ .

## 7.4 Gravitinos and dark matter

Gravitinos are an inevitable consequence of supersymmetry and General Relativity, and there are strict constraints on their mass in the cosmological models with a standard thermal history and an R-symmetry guaranteeing the existence of a lightest supersymmetry particle (LSP) [46]. Even when unstable, they cause trouble either by decaying after nucleosynthesis and photodissociating light elements, or by decaying into the LSP. The result is a constraint on the reheat temperature  $T_{\text{rh}}$  in order to suppress the production of gravitinos. The relic abundance of thermally produced gravitinos is approximately

$$Y_{\frac{3}{2}} \simeq 2.4 \times 10^{-12} \omega_{\tilde{G}} \left( \frac{T_{\text{rh}}}{10^{10} \text{ GeV}} \right), \quad (7.30)$$

where gravitinos are taken much more massive than the other superparticles, and  $\omega_{\tilde{G}}$  is a factor taking into account the variation in the predictions. In recent literature it has taken the value 1.0 [47, 48] and 0.6 [13]. The LSP density parameter arising from

a particular relic abundance in the MSSM is

$$\Omega_{\text{LSP}} h^2 \simeq 2.8 \times 10^{10} \frac{m_{\text{LSP}}}{100 \text{ GeV}} Y_{\frac{3}{2}}. \quad (7.31)$$

The LSP density parameter from thermally produced gravitinos is therefore

$$\Omega_{\text{LSP}} h^2 \simeq 6 \times 10^{-2} \omega_{\tilde{G}} \frac{m_{\text{LSP}}}{100 \text{ GeV}} \left( \frac{T_{\text{rh}}}{10^{10} \text{ GeV}} \right), \quad (7.32)$$

In our model, we will see that the gravitinos generated by the first stage of reheating, or by non-thermal production from decaying long-lived scalars [49], are diluted by a period of thermal inflation. The constraint therefore applies to reheating after thermal inflation.

### 7.5 Thermal inflation in the $h$ -vacuum

In this section we continue with the assumption that the universe exits inflation into the  $h$ -vacuum. As the temperature falls, eventually soft terms in the potential become comparable to thermal energy density, and the universe can seek its true ground state, which we established in Section 5 was  $\chi = \pi/2$ , the  $\phi$ -vacuum. This leads to a second period of inflation, akin to the complementary modular inflation model of Ref. [50]. Unlike this model, we will see that reheating temperature is high enough to regenerate an interesting density of gravitinos, and also to allow baryogenesis by leptogenesis.

At zero temperature the difference in energy density between the  $h$ -vacuum and the  $\phi$ -vacuum is (see Eqs. (5.25), (5.26))

$$\Delta V_{\text{eff}}^0 \simeq s_c^2 \left( 1 + 2 \frac{\lambda_3^2}{\lambda_1^2} \right)^{\frac{1}{2}} \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right)^2 4Q q_\phi^2 \left( 1 - \frac{q_{H_1}^2}{q_\phi^2} \frac{\lambda_1}{\lambda_3} \right). \quad (7.33)$$

Defining an effective SUSY-breaking scale

$$m_{\text{sb}} = \left( \frac{m_{\frac{3}{2}} g'^2}{16\pi^2} \right) q_\phi \sqrt{Q}, \quad (7.34)$$

we see that a period of thermal inflation [51] starts at

$$T_i \simeq \left( \frac{30}{g_{\text{eff}} \pi^2} s_c^2 m_{\text{sb}}^2 \right)^{\frac{1}{4}}. \quad (7.35)$$

Using the CMB normalisation for  $N$  e-foldings of standard hybrid inflation,  $(s_c/m_{\text{P}}) \simeq 3 \times 10^{-3}$  (dropping the unimportant dependence on  $N$ ), and the MSSM value for the degrees of freedom  $g_{\text{eff}} = 915/4$ , we have

$$T_i \simeq 1.0 \times 10^9 \left( \frac{m_{\text{sb}}}{1 \text{ TeV}} \right)^{\frac{1}{2}} \text{ GeV}. \quad (7.36)$$

Thermal inflation continues until the quadratic term in the thermal potential  $q_\phi^2 g'^2 T^2 |X|^2$  becomes the same size as the negative soft mass terms  $m_X^2 |X|^2$ . Hence the transition which ends thermal inflation takes place at  $T_e \sim m_{\text{sb}}$ , and the number of e-foldings of thermal inflation is

$$N_\theta \simeq \frac{1}{2} \ln \left( \frac{s_c}{m_{\text{sb}}} \right) \simeq 17, \quad (7.37)$$

taking  $m_{\text{sb}} \sim 1$  TeV. Thus any gravitinos will be diluted to unobservably low densities, as will any baryon number generated prior to thermal inflation.

There is another period of reheating as the energy of the modulus  $X$  is converted to particles. Around the true vacuum, the  $X$  is mostly Higgs, and so its large amplitude oscillations will be quickly converted into the particles of the MSSM in much less than an expansion time, and the vacuum energy will be efficiently converted into thermal energy. With the assumption of complete conversion of vacuum energy into thermal energy, the reheat temperature following thermal inflation will be

$$T_{\text{rh3}} = \left( \frac{30}{g_{\text{eff}} \pi^2} \Delta V_{\text{eff}}^0 \right)^{\frac{1}{4}} \simeq T_i. \quad (7.38)$$

This reheating regenerates the gravitinos, and we may again apply the gravitino constraint Eq. (7.32), finding

$$\Omega_{\text{LSP}} h^2 \simeq 6 \times 10^{-3} \omega_{\tilde{G}} \frac{m_{\text{LSP}}}{100 \text{ GeV}} \left( \frac{m_{\text{sb}}}{1 \text{ TeV}} \right)^{\frac{1}{2}}. \quad (7.39)$$

We can convert the relic density into a constraint on the gravitino mass, requiring that the LSP density is less than or equal to the observed dark matter abundance,  $\Omega_{\text{dm}} h^2 \simeq 0.1$ , obtaining

$$m_{\frac{3}{2}} \lesssim \frac{5 \times 10^4}{g'^2 q_\phi \sqrt{Q}} \left( \omega_{\tilde{G}} \frac{m_{\text{LSP}}}{100 \text{ GeV}} \right)^{-2} \text{ TeV}. \quad (7.40)$$

Hence this class of models requires a high gravitino mass in order to saturate the bound and generate the dark matter.

We can be a bit more precise if we use the phenomenological relations derived in Section 6. Firstly, in order to fit  $\mu_h$  we have from Eq. (6.1)

$$q_\phi^2 g'^2 \simeq \frac{\lambda_1}{\lambda_3}, \quad (7.41)$$

while we can derive a phenomenological formula for the LSP mass from Table 2

$$m_{\text{LSP}} \simeq 3.3 \times 10^{-3} m_{\frac{3}{2}}. \quad (7.42)$$

Hence

$$m_{\frac{3}{2}} \lesssim 300 \left( \frac{1}{\omega_{\tilde{G}}^2} \frac{q_\phi}{\sqrt{Q}} \frac{\lambda_3}{\lambda_1} \right)^{\frac{1}{3}} \text{ TeV}, \quad (7.43)$$

with the inequality saturated if the gravitino decays supply all the dark matter.

In the case where the dark matter consists of LSPs derived from gravitino decay, we can derive a range of acceptable values for the gravitino mass, as we have a constraint (7.14) on  $\frac{\lambda_3}{\lambda_1}$  from requiring the exit to a false  $h$ -vacuum. Hence, in order for gravitino-derived LSPs in this model to comprise all the dark matter, we have

$$\left( \frac{1}{\omega_{\tilde{G}}^2} \frac{q_\phi}{\sqrt{Q}} \frac{q_{H_1}^2}{q_\phi^2} \right)^{\frac{1}{3}} \lesssim \frac{m_{\frac{3}{2}}}{300 \text{ TeV}} \lesssim \left( \frac{1}{\omega_{\tilde{G}}^2} \frac{q_\phi}{\sqrt{Q}} \right)^{\frac{1}{3}}. \quad (7.44)$$

For example, taking  $q_L = 0$  as in Section 6, and recalling the range of the theoretical predictions  $0.6 \lesssim \omega_{\tilde{G}} \lesssim 1.0$ , we find that  $m_{\frac{3}{2}}$  is independent of  $q_E$  and in the range

$$100 \lesssim m_{\frac{3}{2}} \lesssim 430 \text{ TeV}. \quad (7.45)$$

Interestingly, a Higgs with mass near 125 GeV also demands a high gravitino mass. In order to fit the central value of  $\delta a_\mu$  we require a gravitino mass of 60 TeV, which would require  $\omega_{\tilde{G}} \simeq 2$ , or another source of dark matter.

## 7.6 Cosmic string formation and constraints

The breaking of the  $U(1)''$  gauge symmetry at the end of thermal inflation results in the formation of cosmic strings [52, 53, 54]. The string tension in models with flat directions is much less than the naive calculation, as the potential energy density in core the string is of order  $\Delta V \sim s_c^2 m_{\text{sb}}^2$  rather than  $M^4$ . The vacuum expectation of the modulus field defined in Section 7.2 is still  $X_0 \sim s_c$ , so as a rough approximation we can therefore take the potential as

$$V_{\text{string}} \sim \frac{m_{\text{sb}}^2}{s_c^2} (X^2 - X_0^2)^2. \quad (7.46)$$

showing that there is an effective scalar coupling of order  $(m_{\text{sb}}^2/s_c^2)$ . The string tension is approximately

$$\mu \simeq 2\pi B \left( \frac{m_{\text{sb}}^2}{q_\phi^2 g'^2 s_c^2} \right) \frac{2M^2}{\lambda_1}, \quad (7.47)$$

where  $B$  is a slowly varying function of its argument, with [55]

$$B(\beta) \simeq 2.4/\ln(2/\beta), \quad (\beta < 10^{-2}). \quad (7.48)$$



Hence, for  $q_\phi^2 g'^2 = 2$ ,  $s_c = 3 \times 10^{-3} m_{\text{P}}$ , and  $m_{\text{sb}} = 1 \text{ TeV}$  as above,

$$B \simeq 0.04, \quad (7.49)$$

demonstrating that the string tension is more than an order of magnitude below its naive value  $4\pi s_c^2$ , which reduces the CMB constraint on this model. Hence the string tension in this model is

$$G\mu = \frac{B}{4} \frac{s_c^2}{m_{\text{P}}^2} \sim 10^{-7}, \quad (7.50)$$

well below the 95% confidence limit for CMB fluctuations from strings [56, 57].

There are also other bounds on strings depending on uncertain details about their primary decay channel. Pulsar timing provides a strong bound if the long strings lose a significant proportion of energy into loops with sizes above a light year or so (smaller loops radiate at frequencies to which pulsar timing is not very sensitive). In this case recent European Pulsar Timing Array data [58] can be used to place a conservative upper bound of  $G\mu < 5.3 \times 10^{-7}$  [59] for strings with a reconnection probability of close to unity (as is the case in field theory), and loops formed with a typical size of about  $10^{-5}$  of the horizon size. Future experiments will place tighter (but still model-dependent) bounds [59, 60]. For example, the Large European Array for Pulsars (LEAP) will be two orders of magnitude more sensitive than EPTA [61] and will be able to detect the gravitational radiation from the loops in this model if they are large enough to radiate into the LEAP sensitivity window. Current string modelling [62] indicates this is likely if loop production is significant.

Strings may also produce high energy particles, whose decays can produce cosmic rays over a very wide spectrum of energies. If  $f_{\text{cr}}$  is the fraction of the energy density going into cosmic rays, then the diffuse  $\gamma$ -ray background provides a limit [53]  $G\mu \lesssim 10^{-10} f_{\text{cr}}^{-1}$ . Given that the strings in our model contain a large Higgs condensate, we would expect that all particles produced by the strings would end up as Standard Model particles or neutralinos. Thus we require that the decays are primarily gravitational in order to avoid the cosmic ray bound.

## 7.7 Baryogenesis

Baryon asymmetry requires baryon number (B) violation, C violation, and CP violation [63]. In common with the standard model, our model has C violation and sphaleron-induced B violation. It can also support CP-violating phases in the neutrino Yukawa couplings. In [7], it was pointed out that leptogenesis [64] was natural in the model, provided that the reheat temperature is greater than about  $10^9 \text{ GeV}$ .

As we saw in Section 7.5, this is the approximate value of the reheat temperature after thermal inflation, and so we require at least one right-handed neutrino which is

sufficiently light to be generated in the reheating process, i.e. with a mass less than around  $10^9$  GeV. The baryogenesis in our model should therefore be similar to that of Ref. [65].

In sAMSB, the light scalars are weakly coupled to the Higgs (the stops are both at the TeV scale), and so the electroweak phase transition is a crossover [66]. This means that there is no conventional electroweak baryogenesis (see e.g. Ref. [67] for a recent review).

## 8. Conclusions

The sAMSB model, as described here, is in our opinion the most attractive way of resolving the tachyonic slepton problem of anomaly mediated supersymmetry breaking. The low energy spectrum is similar to that of regions of CMSSM or MSUGRA parameter space, but with characteristic features, most notably a wino LSP. We have seen that, while it is possible to obtain a light SM-like Higgs with a mass of 125 GeV, this requires fine-tuning and also results in a suppression of the supersymmetric contribution to  $a_\mu$ , so that the current theoretical prediction for  $a_\mu$  in our model is about  $3\sigma$  below the experimental value.<sup>3</sup>

Moreover, to produce a Higgs of over 120 GeV, we need to increase the gravitino mass to over 80 TeV. If the gravitino mass is over 100 TeV we can use wino LSPs derived from gravitino decays to account for all the dark matter.

Assuming that the  $U(1)'$  introduced to solve the tachyonic slepton problem is broken at a high scale,  $M$ , we have seen that sAMSB naturally realises F-term hybrid inflation. The universe may exit the inflationary era into a vacuum dominated by large vevs for the MSSM Higgs fields,  $h_{1,2}$ , with the true vacuum with unbroken  $SU(3) \otimes SU(2) \otimes U(1)_Y$  (above the electroweak scale) attained only after a later period of approximately 17 e-foldings of thermal inflation.

The thermal inflation reduces the number of e-foldings of high-scale inflation to about 40, and hence the spectral index of scalar CMB fluctuations is reduced to about 0.975, within about  $1\sigma$  of the WMAP7 value. Cosmic strings are formed at the end of thermal inflation, with a low mass per unit length, satisfying observational bounds provided their main decay channel is gravitational, and the typical size of string loops at formation is about  $10^{-5}$  of the horizon size, or so small that they radiate at a frequency below  $1 \text{ yr}^{-1}$ , to which pulsar timing is not sensitive. The Large European Array for Pulsars will be two orders of magnitude more sensitive, and be capable of closing the window in the loop size at  $10^{-5}$  of the horizon, or detecting the gravitational radiation.

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<sup>3</sup>This tension has also been noted in the CMSSM [68], underlining the importance of an independent experimental measurement.

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